DRONACHARYA College of Engineering

INTELLIGENT SYSTEMS (CSE-303-F)

Section C

Fuzzy Reasoning

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Bivalent and Multivalent Logics

- Bivalent (Aristotelian) logic uses two logical values – true and false.
- Multivalent logics use many logical values – often in a range of real numbers from 0 to 1.
- Important to note the difference between multivalent logic and probability – P(A) = 0.5 means that A may be true or may be false – a logical value of 0.5 means both true and false at the same time.

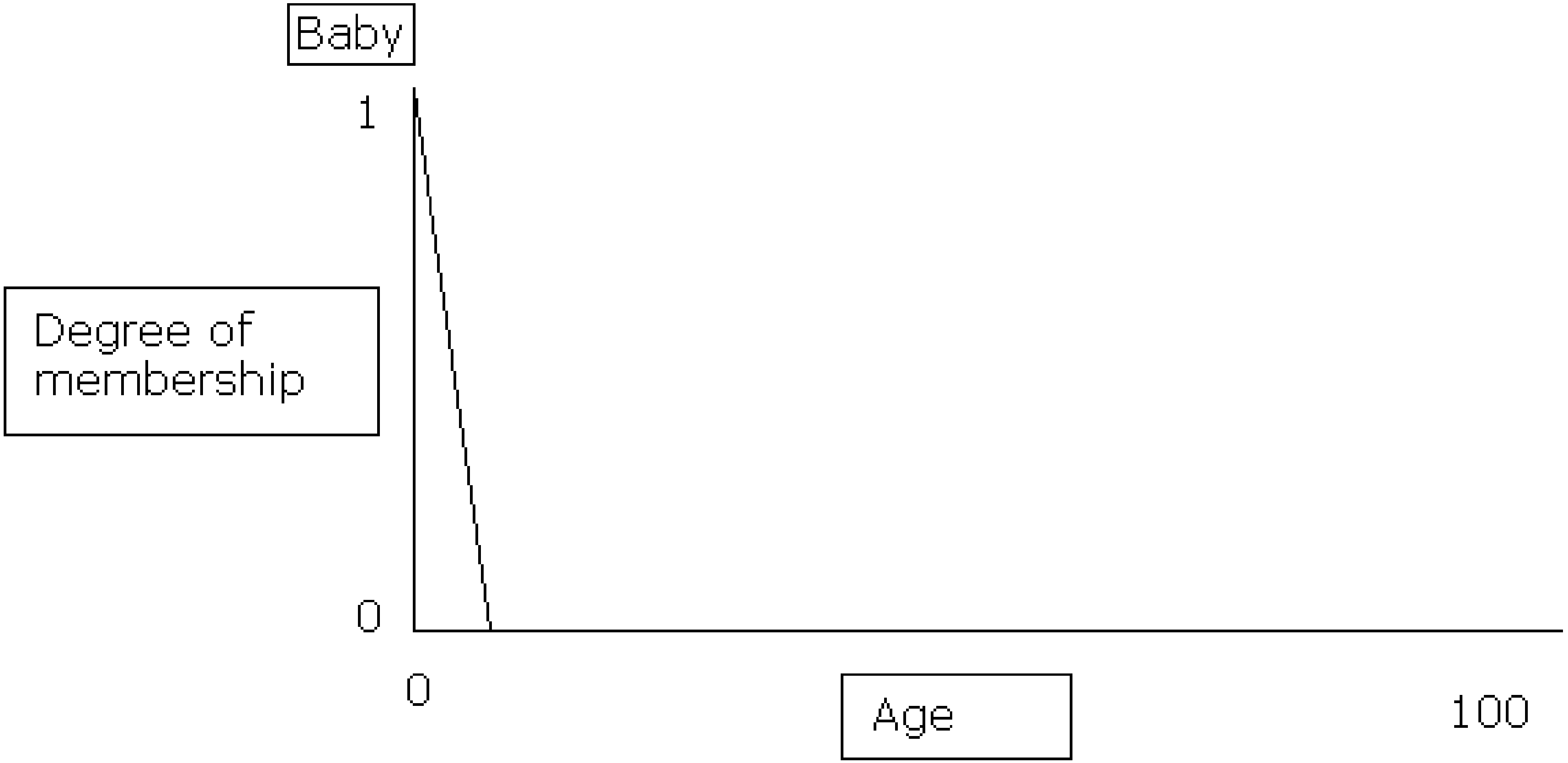
Linguistic Variables

- Variables used in fuzzy systems to express qualities such as height, which can take values such as "tall", "short" or "very tall".
- These values define subsets of the universe of discourse.

Fuzzy Sets

- A crisp set is a set for which each value either is or is not contained in the set.
- For a fuzzy set, every value has a membership value, and so is a member to some extent.
- The membership value defines the extent to which a variable is a member of a fuzzy set.
- The membership value is from 0 (not at all a member of the set) to 1.

Membership Functions



Crisp Set Operators

- Not A – the complement of A, which contains the elements which are not contained in A.
- $A \cap B$ – the intersection of A and B, which contains those elements which are contained in both A and B.
- $A \cup B$ – the union of A and B which contains all the elements of A and all the elements of B.
- Fuzzy sets use the same operators, but the operators have different meanings.

Fuzzy Set Operators

- **Fuzzy set operators can be defined by their membership functions**
  - $M_{\neg A}(x) = 1 - M_A(x)$
  - $M_{A \cap B}(x) = MIN(M_A(x), M_B(x))$
  - $M_{A \cup B}(x) = MAX(M_A(x), M_B(x))$
- **We can also define containment (subset operator):**
  - $B \subset A$ iff $\forall x (M_B(x) \le M_A(x))$

Hedges

- A hedge is a qualifier such as “very”, “quite”, “somewhat” or “extremely”.
- When a hedge is applied to a fuzzy set it creates a new fuzzy set.
- Mathematic functions are usually used to define the effect of a hedge.
- For example, “Very” might be defined as:
  - $M_{VA}(x) = (M_A(x))^2$

Fuzzy Logic (1)

- A nonmonotonic logical system that applies to fuzzy variables.
- We use connectives defined as:
  - $A \vee B \equiv \mathrm{MAX}(A, B)$
  - $A \wedge B \equiv \mathrm{MIN}(A, B)$
  - $\neg A \equiv 1 - A$
- We can also define truth tables:

A	B	$A \vee B$
0	0	0
0	0.5	0.5
0	1	1
0.5	0	0.5
0.5	0.5	0.5
0.5	1	1
1	0	1
1	0.5	1
1	1	1

Fuzzy Inference (1)

- Inference is harder to manage.
- Since:

  $A \rightarrow B \equiv \neg A \vee B$
- Hence, we might define fuzzy inference as:

  $A \rightarrow B \equiv \text{MAX}((1 - A), B)$
- This gives the unintuitive truth table shown on the right.
- This gives us $0.5 \rightarrow 0 = 0.5$, where we would expect $0.5 \rightarrow 0 = 0$

A	B	A → B
0	0	1
0	0.5	1
0	1	1
0.5	0	0.5
0.5	0.5	0.5
0.5	1	1
1	0	0
1	0.5	0.5
1	1	1

Fuzzy Inference (2)

- An alternative is Gödel implication, which is defined as:

  $A \rightarrow B \equiv (A \leq B) \vee B$

- This gives a more intuitive truth table.

A	B	A → B
0	0	1
0	0.5	1
0	1	1
0.5	0	0
0.5	0.5	1
0.5	1	1
1	0	0
1	0.5	0.5
1	1	1

Fuzzy Inference (3)

- Mamdani inference derives a single crisp value by applying fuzzy rules to a set of crisp input values.

Step 1: Fuzzify the inputs.

Step 2: Apply the inputs to the antecedents of the fuzzy rules to obtain a set of fuzzy outputs.

Step 3: Convert the fuzzy outputs to a single crisp value using defuzzification.

Fuzzy Rules

- A fuzzy rule takes the following form:

  IF A op x then B = y
- op is an operator such as >, < or =.
- For example:

  IF temperature > 50 then fan speed = fast

  IF height = tall then trouser length = long

  IF study time = short then grades = poor

Fuzzy Expert Systems

- A fuzzy expert system is built by creating a set of fuzzy rules, and applying fuzzy inference.
- In many ways this is more appropriate than standard expert systems since expert knowledge is not usually black and white but has elements of grey.
- The first stage in building a fuzzy expert system is choosing suitable linguistic variables.
- Rules are then generated based on the expert's knowledge, using the linguistic variables.

Neuro-Fuzzy Systems

- A fuzzy neural network is usually a feed-forward network with five layers:

1. Input layer – receives crisp inputs
2. Fuzzy input membership functions
3. Fuzzy rules
4. Fuzzy output membership functions
5. Output layer – outputs crisp values